Photoelectron trapping in quadrupole and sextupole magnetic fields

L. F. Wang,* H. Fukuma, S. Kurokawa, and K. Oide
High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

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A photoelectron-trapping phenomenon has been found in the simulation of the photoelectron cloud. It is found that the photoelectrons can be trapped in the quadrupole and sextupole magnetic fields for very long time until it longitudinally drifts out of the magnets, even a long bunch train separation is not sufficient to clear up the photoelectrons. Therefore, such a kind of long time trapped photoelectron cloud can cause coupled bunch instability. The trapping phenomenon is strongly beam dependent, especially on the bunch length. There is no such kind of trapping if the positron beam does not disturb the photoelectrons during the whole process. There is also no trapping for positron bunch with bunch length longer than the period of the photoelectron's gyration motion at the mirror points. The trapping is a mirror field trap which is caused by beam disturbance. The trapping phenomenon and mechanism will be presented in detail.

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I. INTRODUCTION

A blowup of the vertical beam size is observed in the KEKB positron ring (LER) [1] and it is one of the serious problems limiting the luminosity of KEKB. A numerical method has been applied to study the electron-cloud effects in different magnetic fields. Among them, a trapping phenomenon was found in quadrupole and sextupole magnets. The trapped electron cloud density near the beam is low, which means the trapped electron cloud does not contribute to the blowup of the positron bunch according to Ohmi and Zimmermann's model [2]. However, it may cause coupled bunch instability due to its long trapping time. Experimental studies in KEKB LER show that the electron cloud can cause coupled bunch instability even with the solenoid in the drift region, which means that the cloud inside the magnets is the source of the instability. Among them, the cloud in the dipole magnet can decay quickly after the passage of positron bunches. On the other hand, the electron cloud in quadrupole and sextupole magnets has a long decay time. Therefore, the cloud in quadrupole and sextupole magnets is the main source of the coupled bunch instability. The trapping phenomenon and mechanism is presented in detail.

II. TRAPPING PHENOMENON

A three-dimensional (3D) particle-in-cell program [3–5] was developed to study the effects of various magnetic fields on the photoelectron formation, distribution, space charge effect, multipacting, and so on. It is assumed that the positron bunch is longitudinally divided into a number of slices according to the Gaussian distribution in simulation. Such slices interact with photoelectrons transversely and oscillate according to the transfer matrix of the linear optics. Photoelectrons are emitted when the positron bunch slices pass through a beam pipe with period length L. A photoelectron yield of 0.1 is also assumed in the simulation and 30% of the photoelectrons are produced by the reflective photons. The center of photoelectron energy distribution is at 5 eV with an rms (root mean square) energy spread of 5 eV. The secondary emission also is included. In our simulation, the photoelectrons are represented by macroparticles, which move in three-dimensional space under the space charge force of the photoelectron cloud [6], positron beam force, and magnetic field force. The longitudinal boundary of the photoelectron motion is periodical. Therefore, there is no electron loss due to the longitudinal boundary. The average velocity of the electron cloud in positron beam direction, which is very small due to the strong transverse magnet fields, can explain the magnet length effect.

The parameters used in the simulation are shown in Table I.

Figure 1 shows a typical simulation result of the photoelectron volume density for a passage of 200 bunches with a bunch spacing of four rf buckets followed by a long train separation in dipole, normal quadrupole, and sextupole magnetic fields. The electron-cloud density is almost constant during the bunch train separation in both quadrupole and sextupole magnets because most of the electrons have been trapped by the magnetic fields. More than 80% of the electrons at the end of bunch train can be trapped. However, the

TABLE I. Simulation parameters.

Variable	Symbol	Value
Ring circumference	C	3016.26 m
Rf bucket length	s_{rf}	0.589 m
Bunch spacing	s_b	7.860 ns
Bunch population	N	3.3×10^{10}
Average horizontal/vertical betatron function	β_y/β_x	10/10 m
Horizontal emittance	ε_x	1.8×10^{-8} m
Vertical emittance	ε_y	3.6×10^{-10} m
Rms bunch length	σ_l	4 mm
Chamber diameter	$2R$	100 mm
Rf harmonic number	h	5120

*On leave from IHEP, Beijing.
Email address: wanglf@post.kek.jp

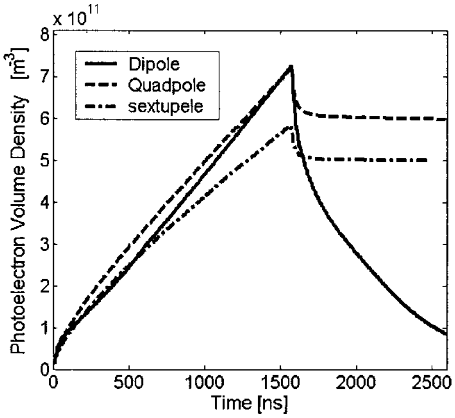


FIG. 1. Photoelectron average volume density in different magnet fields as a function of time for a bunch train with 200 bunches spaced by 7.86 ns and followed by a long separation.

cloud density in the dipole magnet decreases continuously with a short decay time during the bunch train separation. Therefore, there is no trapping in the dipole magnet. Figure 2 shows a typical orbit of a trapped electron in the quadrupole field during the bunch train separation with drift time 943 ns. The normal value of the quadrupole field gradient in KEKB LER is 10.3 T/m. A smaller gradient of 0.5 T/m is applied in Fig. 2 in order to show the clear orbit with large gyration radius. The energy of the electron is 5.9 eV. The electron drifts along the field line and is reflected by the strong magnet field, which means the electron spirals in an ever-tighter orbit along the lines of force, converting more and more translational energy into energy of rotation, until its translational velocity vanishes. Then it turns around, still spiraling in the same manner, and moves back in the opposite translational direction. The average orbit over the cyclotron in the transverse plane is the magnetic field line as shown in Fig. 2(b). There is also a very slow average velocity along the beam direction [z direction in Fig. 2(a)], which decides the trapping time. There is a very similar trapping phenomenon in the sextupole magnet. One sample orbit of a trapped electron in the sextupole magnet is shown in Fig. 3. The gradient of the sextupole field is 350 T/m^2.

Some photoelectrons also have very long lifetime during the bunch train separation in the periodic solenoid case. There is no electron cloud in the chamber center when the periodic solenoids are arranged with the same current direction in the coil, which is called the equal polarity configuration. It is better than the configuration with alternating current direction [4]. In the equal polarity case, there are less than 1% of electrons with long lifetime. Among them, some of electrons are trapped by the solenoid magnetic field. Other electrons move along the longitudinally periodical magnetic field line forever and never turn around. Therefore, the trapping phenomenon is not dominant in the solenoid case. We do not discuss it in the present study.

The positron beam force plays an important role in the trapping phenomenon. There would be no such trapping in quadrupole and sextupole magnetic fields if the positron beam did not perturb the photoelectron during the whole process. The trapping phenomenon depends on the positron bunch length, bunch current, and bunch spacing, which affect the interaction of the photoelectron and positron beam. The trapping phenomenon occurs during both bunch train and bunch train separation. The positron beam force disturbs the photoelectron during the bunch train and brings on photoelectron trapping during the bunch train separation. The characters of the trapping phenomenon can be summarized as follows.

(1) The trapping phenomenon occurs in quadrupole and sextupole magnets. More than 80% of the photoelectrons can be trapped in these magnets during the bunch train separation. The longer the bunch train, the more the trapped electron percentage. Periodic solenoid field can also trap less than 1% of photoelectrons. However, there is no such trapping in dipole magnet.

(2) The photoelectron can be trapped in the quadrupole and sextupole magnets for a very long time until it drifts out of the magnet longitudinally.

(3) The trapping phenomenon is strongly beam dependent, especially on the bunch length.

III. TRAPPING MECHANISM

We first describe the motion of a photoelectron in a pure magnetic field and then focus on the effects of the positron beam.

First we consider the case of no electric field, which is almost true for the electron cloud during the bunch train separation, where the space charge potential of the electron cloud is negligible compared with the magnetic potential in normal magnets. Since the direction of magnetic force acting on the electron is perpendicular to the electron velocity, the electron kinetic energy is therefore conserved,

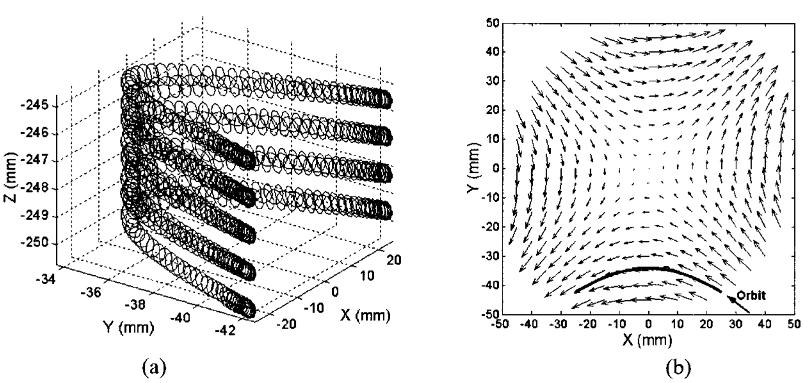


FIG. 2. Orbit of a trapped photoelectron in a normal quadrupole magnet during the bunch train separation. (a) 3D orbit, (b) 2D orbit. Black solid line shows the 2D orbit and black arrow denotes the quadrupole magnetic field.

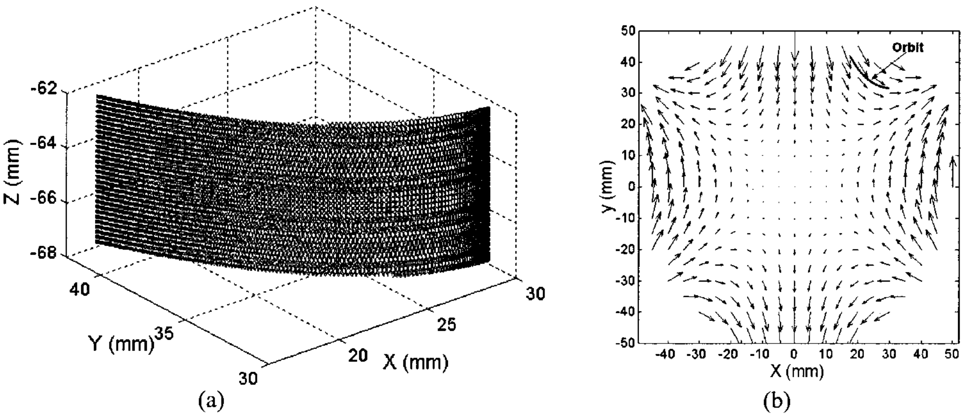


FIG. 3. Orbit of a trapped photoelectron in a normal sextupole magnet during the bunch train separation. (a) 3D orbit, (b) 2D orbit. Black solid line shows the 2D orbit and black arrow denotes the sextupole magnetic field.

$$W=\frac{mv^2}{2}=\text{const.} \tag{1}$$

The motion of the electron in the magnetic field can be regarded as the superposition of the gyration motion around the guiding center and the motion of the guiding center. The gyration motion of the electron is a rapid rotation around the magnetic field line. The motion of the guiding center is the average motion over the gyration motion.

Consider the case in which the magnetic field slowly varies in space. The variation is assumed to be sufficiently slow that the magnetic field at the electron position hardly changes during the cyclotron motion. This is true for our case where the magnetic field is strong except for the central region of the chamber and the electron energy is low, which means a small Larmor radius and a short period. While the period of a spiraling electron changes as the electron moves into regions where the magnetic field is weaker or stronger, the product *TE*, the period *T* times the energy *E*, is almost a constant. It is not an exact constant, but if the rate of change is slow enough, e.g., if the field changes rather slowly, it comes very close. A certain quality, an "adiabatic invariant," is almost kept at a constant value. In a more general way, the action of a system with canonical variables q and p defined by

$$J=\oint p\,dq \tag{2}$$

is a constant under a slow change in an external parameter. Equation (2) represents an integral over one period of the motion. Therefore, for such a quasiperiodic motion, there exist two adiabatic invariations given by [7]

$$J_\perp=\oint mv_\perp\rho_s d\varphi=\frac{4\pi m}{e}\mu_m, \tag{3}$$

$$J_\parallel=\oint mv_\parallel dl, \tag{4}$$

where

$$\mu_m=\frac{mv_\perp^2}{2B} \tag{5}$$

is the magnetic moment, $v_\perp$ is the gyration velocity, $\rho_s=mv_\perp/|e|B$ is the Larmor radius, and $v_\parallel$ is the parallel or longitudinal velocity, which is parallel to the magnetic field. $J_\perp$ and $J_\parallel$ are called the transverse and parallel adiabatic invariations, respectively.

As the guiding center of the electron moves along the field line, which will be explained below, the magnetic field strength at the electron changes. Because the magnetic moment and kinetic energy of the electron are conserved, the kinetic energy of the parallel motion varies according to the relation

$$\tfrac{1}{2}mv_\parallel^2+\mu_m B=\text{const.} \tag{6}$$

Recalling the motion of a pendulum in the earth's gravitation potential, Eq. (6) implies that the guiding center motion along the field line behaves like a particle motion in a magnetic potential energy $\mu_m B$. The magnetic field is a mirror field in the quadrupole and sextupole magnets, in which the magnetic field is weaker at the center and is stronger at both ends of the mirror field line. When the guiding center of the electron moves along the field line from a weaker field region to a stronger field region, the parallel velocity decreases and the gyration velocity increases and the electron is heated. This kind of heating is called adiabatic heating in the plasma field. Therefore, the electron spirals in an ever-tighter orbit because the period of gyration motion and parallel velocity become smaller and smaller. When the electron comes to the point where the parallel velocity vanishes, the electron is reflected. The parallel velocity of the reflected electron is increased when it moves along the field line and gets the maximum value at the weakest field point (mirror point). Then it continues a similar motion along the other side of the mirror point. Such a kind of trap is called a magnetic mirror trap. The motion of the electron in the mirror field is shown in Fig. 4. The trap condition is

$$mv_\parallel^2/2<\mu_m(B_{\max}-B), \tag{7}$$

where $v_\parallel$ is the parallel velocity at position with magnetic field B, $B_{\max}$ is the maximum magnetic field along this field line, which is located near the vacuum chamber wall in our

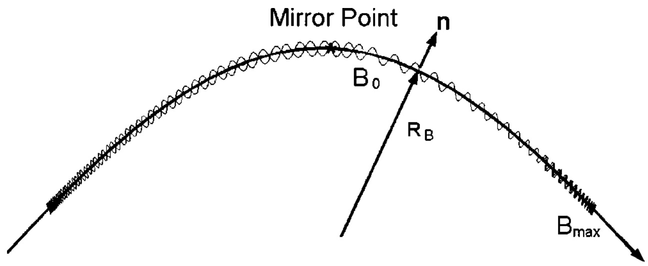


FIG. 4. Motion of an electron in a mirror magnetic field.

case. Note that the trap strongly depends on the electron velocity $v_{\parallel 0}$ and $v_{\perp 0}$. According to Eqs. (1), (3), and (5), the trap occurs if

$$\frac{v_{\perp 0}^2}{v_{\perp 0}^2+v_{\parallel 0}^2}>\frac{B_0}{B_{\max}}, \tag{8}$$

where B_0 is the field at one position with velocity $v_{\parallel 0}$ and $v_{\perp 0}$. The trap condition Eq. (8) can be more conveniently described as

$$\Gamma_{\text{trap}}>1, \tag{9}$$

with the trap factor

$$\Gamma_{\text{trap}}=\frac{F_v}{F_B}=\frac{v_{\perp 0}^2}{v_{\perp 0}^2+v_{\parallel 0}^2}\frac{B_{\max}}{B_0}, \tag{10}$$

where F_v and F_B are the left and right parts of Eq. (8), respectively. When the trap factor Γ_{trap} is bigger than 1, the electron is trapped.

All photoelectrons are emitted from the vacuum chamber wall, where the magnetic field is the strongest along the field line. If the space charge force of the positron bunch and the electron cloud did not disturb the electron during the whole drift process, the trap factor would be a constant value at any time with $\Gamma_{\text{trap}}=\mu_m B_{\max}/W=F_v^{\text{pipe}}=v_\perp^2/(v_\perp^2+v_\parallel^2)|_{\text{at chamber surface}}$ and it is always smaller than 1. Therefore, the electron could not be trapped if there was no other force, for instance, in the beam line of a light source.

Consider F_B, which is only magnetic field dependent. The maximum magnetic field $B_{\max}$ is the same for all field lines, supposing round chamber and constant field gradient. On the other hand, the field at the mirror point is the weakest one along the field line. Therefore, F_B is smaller for the field lines along which the mirror points are near the chamber center. These are the field lines with the azimuth angle around $\pm\pi/4$, $\pm 3\pi/4$ and $\pm\pi/6$, $\pm\pi/2$, $\pm 5\pi/6$ for normal quadrupole and sextupole fields, respectively. The zero azimuth angle is defined as the direction with $y=0$ and $x>0$. All these field lines have strong radial field near the chamber wall, and they are called strong radial field lines in this paper. The electron motion is nonadiabatic along these field lines at the chamber center. The local Larmor radius near the center is larger than the chamber radius. Since the B field vanishes at the chamber center, the magnetic moment μ_m is not preserved. Because of this, the adiabatic invariant does not guarantee that the electron trapped outside of the chamber center will still be trapped after passing through the nonadiabatic region. Figure 5 shows the orbit of an electron in the strong radial field region. The electron, which satisfies the trap condition at the initial position far from the central region, drifts along the strong radial field lines in a special way and finally meets the chamber wall. The special motion in the nonadiabatic region (central region of the chamber) is very clear.

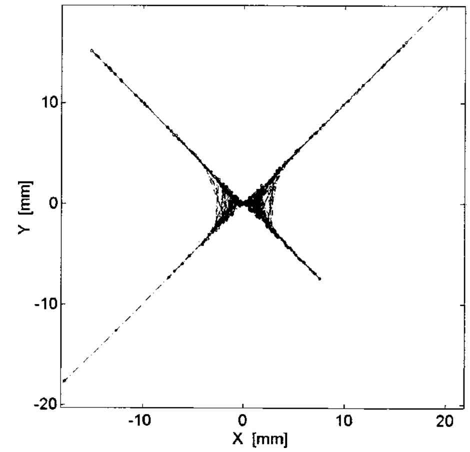


FIG. 5. Electron orbit around the strong radial field lines.

F_v is governed by the photoelectron-beam interaction, which is dependent on both beam force and magnetic field. The beam force plays a very important role in the photoelectron-beam interaction. It can accelerate the photoelectron and change the distribution of photoelectron cloud. The trap condition Eq. (9) can be satisfied due to the effect of the positron beam force. The beam potential of a Gaussian bunch can be expressed as

$$U=\frac{eN}{4\pi\varepsilon_0}\int_0^\infty \frac{\exp\left[-\frac{x^2}{2\sigma_x^2+t}-\frac{y^2}{2\sigma_y^2+t}\right]}{(t+2\sigma_x^2)^{1/2}(t+2\sigma_y^2)^{1/2}}dt, \tag{11}$$

where N is the number of particles per bunch and $\sigma_{x,y}$ is the transverse beam size. If an electron remained at its location during the bunch passage, the largest beam kick is about 30 keV for the $N=3.3\times 10^{10}$, $\sigma_x=0.4$ mm, $\sigma_y=0.06$ mm case, which are the typical parameters of KEKB LER. Therefore, the beam force can effectively change the photoelectron energy, which is very small ($\sim$10 eV) when one photoelectron is emitted. The more important point is that the acceleration of the electron strongly depends on the magnet field distribution. As a result, the energy distribution of photoelectron, F_v, is also strongly magnetic field dependent. Hence, F_v depends on the field line shape and the positron beam related parameters such as bunch length, bunch spacing, and bunch current.

The strong radial field lines cannot usually trap any electrons because photoelectrons moving along these field lines can be easily accelerated in the parallel direction by the positron bunch due to the parallel magnetic field with the bunch electric field. Therefore, these electrons can receive enough parallel energy, which then reduces the quantity of F_v, and

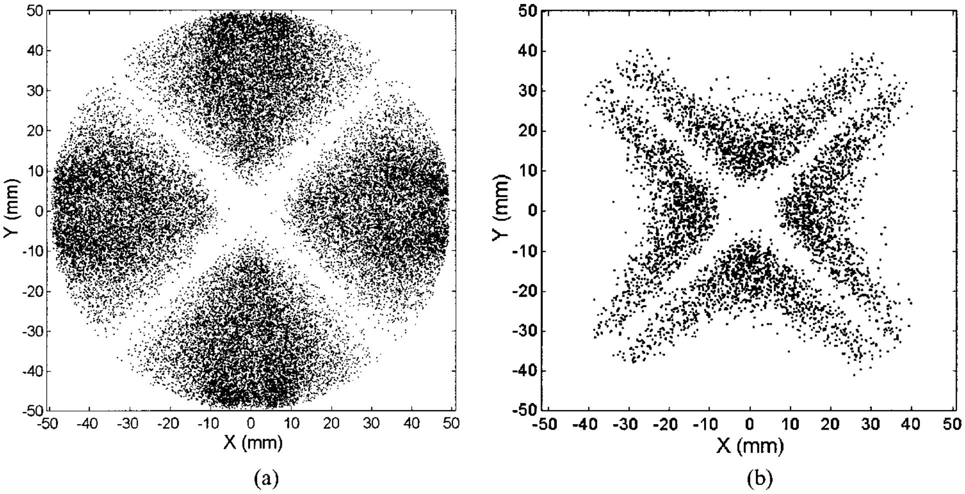


FIG. 6. Trapped photoelectron distribution in the transverse plane in a normal quadrupole magnet with field gradient 10.3 T/m during the bunch train separation for (a) 4 mm and (b) 4 cm bunch length cases.

quickly drift through the magnetic field line until it meets the chamber wall, although F_B is small for these field lines. The additional nonadiabatic region in the chamber center, as explained above, causes larger losses.

According to Eqs. (9) and (10), a photoelectron could be trapped if its kinetic energy of gyration motion increases. The electron can receive transverse energy around the mirror point where the electric field direction of the positron bunch is in the gyration motion plane. However, a short bunch is required for the electron to efficiently receive transverse energy because the effect of a long positron bunch on the transverse energy can cancel over many periods of gyration motion. Therefore, a short positron bunch, when compared with the cyclotron period at the mirror point, is very effective to increase the photoelectron energy distribution F_v by increasing the kinetic energy of the gyration motion and then can cause the trapping of the photoelectrons. In the case of the short positron bunch, electrons can get more kinetic energy of the gyration motion around the mirror points due to the high beam potential at that point and the short interaction time. A long positron bunch has less average effect on the transverse energy of the photoelectron for all the field lines. Therefore, there the effect is weak on the trap of the photoelectron. The trapping requirement for the positron bunch length can be described as

$$\sigma_l < \frac{2\pi c m}{eB}, \qquad (12)$$

where B is the field at the mirror point. Equation (12) can be written in a more convenient way as $\sigma_l\,(\mathrm{mm}) < 10.7/[B\,(\mathrm{T})]$, which means the positron bunch length should be shorter than 10.7 mm for a field line with 1 T magnetic field at the mirror point.

The bunch length of KEKB LER is 4 mm. The cyclotron period of one electron in a 0.5 T magnetic field is 0.07 ns, which is about five times of the bunch length. The maximum field in the quadrupole and sextupole magnets of KEKB LER is 0.52 and 0.44 T, respectively. Note that the magnetic field is proportional to r and r^2 in quadrupole and sextupole magnets, respectively. Here r is the distance to the chamber center. Therefore, the bunch length is shorter than the cyclotron period in all regions of theses two magnets.

When the bunch length increases, the photoelectrons can only be trapped along the field lines on which the cyclotron period at mirror points is shorter than bunch length. The mirror points of these field lines are near the chamber center. Therefore, the longer the bunch length, the smaller the possible trapping area. Figure 6 shows the photoelectron distribution in a quadrupole magnet during the bunch train separation for different bunch length cases. The bunch length effect on the trap region is very clear. Figure 7 shows the normalized trap factor distribution of the electron cloud at different times within the bunch train and at the bunch train separation in quadrupole magnets. The trap factor is normalized by the number of total electrons at each moment. Therefore, the integral of the trap factor, which is the area under the trap factor line in Fig. 7, is 1. The bunch length is 4 mm. IB in the figure is the number of positron bunches which pass through the electron cloud. For example, IB = 10 means the time after ten positron bunches passages. The percentage of the trapped electrons at different times is also shown in the legend of Fig. 7. As more and more bunches pass through the

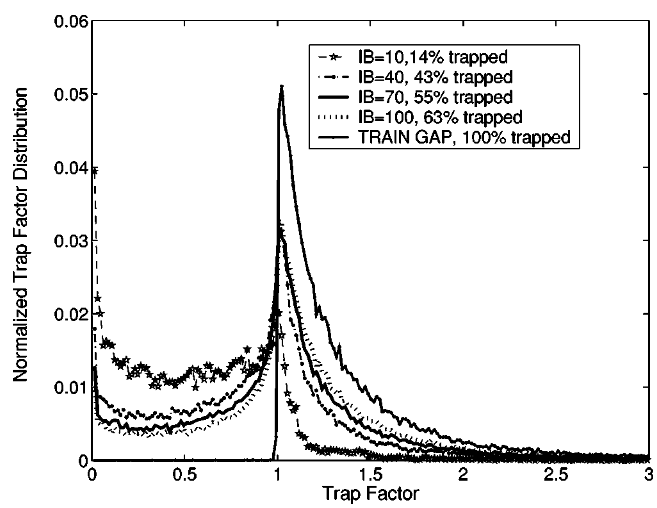
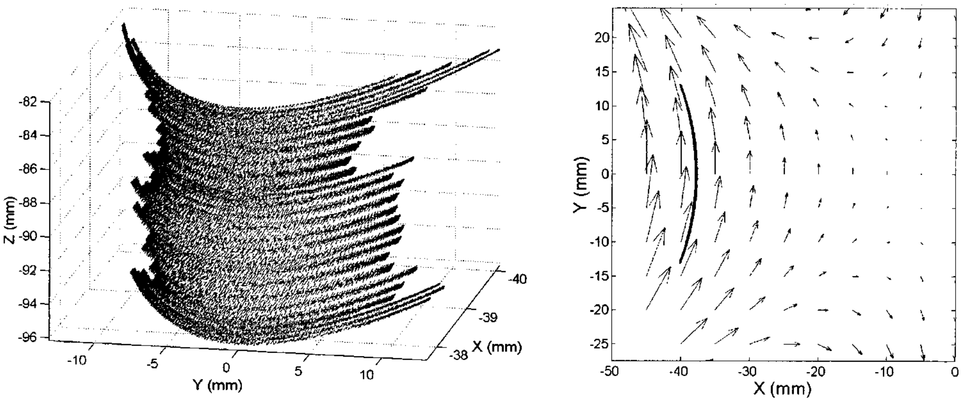


FIG. 7. Trap factor distribution of the electron cloud at different times in a normal quadrupole magnet for 4 mm bunch length.



(a) (b)

FIG. 8. Orbit of a photoelectron in a normal sextupole magnet within the bunch train. (a) 3D orbit, (b) 2D orbit. Solid line shows the 2D orbit and arrows denote the sextupole magnetic field.

electron cloud, more and more photoelectrons will be trapped. The positron beam effect on trapping is very clear. After the passage of the last bunch in the bunch train, the nontrapped photoelectrons disappear quickly and the trapped electrons are left. Therefore, all the trap factors of the photoelectrons during the bunch train separation are larger than 1 as shown in Fig. 7. It agrees very well with the Eq. (9).

There is a similar trapping mechanism within the bunch train where the photoelectron is disturbed by the positron bunch during the bunch passage and drifts under the magnetic force as shown above. The bunch spacing 8 ns is much longer than the bunch length 0.01 ns. Therefore, the behavior of an electron within the bunch train is similar to that during the bunch train separation due to the same trapping mechanism. The positron beam force changes F_v, and hence the location of the reflective points. Figure 8 shows the electron orbit in the sextupole magnet during a bunch train with 100 bunches (786 ns).

IV. ORBIT OF THE GUIDING CENTER

The orbit surface of the drift motion in general magnetic field is given by [7]

$$\mathbf{A}^* = \mathbf{A} + \frac{m v_\parallel}{eB}\mathbf{B} = \text{const.}, \tag{13}$$

where $\mathbf{A}$ is the vector potential of the magnetic field. For the translationally symmetric quadrupole and sextupole field, the orbit of the guiding center is given by $A_z = \text{const}$. Therefore, the orbit of the guiding center is the magnetic field line which is clearly shown in Figs. 2(b) and 3(b) with $A_z = A_2(x^2 - y^2)$ for the normal quadrupole field and $A_z = A_3(x^3 - 3xy^2)$ for the normal sextupole field, respectively.

Besides the movement along the field line, the guiding center also moves along the z direction, which is the positron beam direction, as shown in Figs. 2(a) and 3(a). The drift along the z direction is due to the magnetic field gradient drift and the centrifugal force. The gradient of the magnet field causes the electron drift in the direction perpendicular to the magnetic field $\mathbf{B}$ and the gradient of the field ∇B [8],

$$\overline{\boldsymbol{v}}_{\text{grad}} = \frac{m v_\perp^2}{2eB^3}\mathbf{B} \times \nabla B, \tag{14}$$

where e is the charge of an electron with $e < 0$ in this section. There are opposite drift directions for the electron and positron. This is called the gradient $\mathbf{B}$ drift. The field line is curved in the quadrupole and sextupole magnets. The gradient can be divided into two components: the tangential and normal gradient relative to the field line. The tangential gradient of the field does not cause the gradient drift according to Eq. (14) because it is parallel to the magnetic field. The normal gradient, which is $-B\mathbf{n}/R_B$, causes gradient drift, where $\mathbf{n}$ is the unit vector normal to the field line and R_B is the radius of local curvature of the magnetic field line as shown in Fig. 4. Using Eq. (14), we can get the gradient drift velocity

$$\overline{\boldsymbol{v}}_{\text{grad}} = \frac{\mathbf{n}}{2B\Omega_s} \times \frac{\mathbf{B}}{R_B} v_\perp^2, \tag{15}$$

where $\Omega_s = eB/m$ is the gyration angular velocity. Note that $\Omega_s < 0$ for the electron here.

Now we consider a curved magnetic field line as in the case of the quadrupole and sextupole magnets. The guiding center motion that follows the field line is deflected along the curve and as a result the electron undergoes an inertial force or a centrifugal force perpendicular to the field line,

$$\mathbf{F}_c = \frac{m v_\parallel^2 \mathbf{R}_B}{R_B^2} = \frac{m v_\parallel^2}{R_B}\mathbf{n}. \tag{16}$$

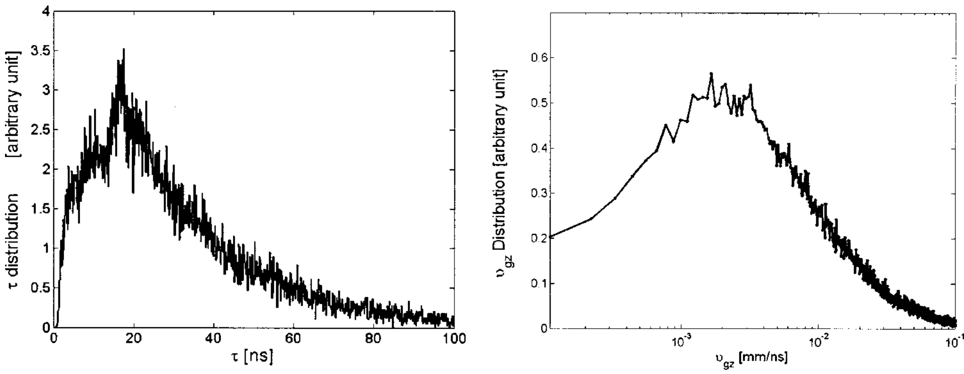
Therefore, the drift velocity of the guiding center due to the force $\mathbf{F}_c$ is

$$\overline{\boldsymbol{v}}_F = \frac{\mathbf{F}_c \times \mathbf{B}}{eB^2} = \frac{\mathbf{n}}{B\Omega_s} \times \frac{\mathbf{B}}{R_B} v_\parallel^2. \tag{17}$$

Combining with Eq. (15), we can get the total drift velocity of the guiding center along the z direction,

$$\overline{\boldsymbol{v}}_{gz} = \frac{\mathbf{n}}{B\Omega_s} \times \frac{\mathbf{B}}{R_B}(v_\parallel^2 + v_\perp^2/2). \tag{18}$$

Note the dependence of v_{gz} on Ω_s in Eq. (18), which is particle charge dependent. Therefore, there is a different drift direction for the positron and electron. All parameters in the above formula are location dependent and the drift velocities



(a) (b)

FIG. 9. (a) Period *t* and (b) average *z*-direction drift velocity of the trapped electron cloud in a quadrupole during the bunch train separation for 4 mm bunch length.

are the average value over the period of gyration motion. Applying the law of kinetic energy conservation and magnetic moment conservation, the above equation can be rewritten as

$$\overline{v_{gz}}=\frac{1}{eR_B}\left(\frac{2W}{B}-\mu_m\right). \tag{19}$$

From Eq. (19), $\overline{v_{gz}}$ is modulated by the field strength B and the radius R_B. Its average value over one period of the parallel motion between the two turning points, which decides the trapping time of the electron, is

$$\bar{\bar{v}}_{gz}=\frac{1}{\tau}\oint\frac{\overline{v_{gz}}}{v_{\parallel}}dl, \tag{20}$$

where τ is the period of the electron parallel motion, which is defined as

$$\tau=\oint\frac{dl}{v_{\parallel}}. \tag{21}$$

The electron shown in Fig. 2 drifts about four periods and 6.2 mm in the z direction during 943 ns. Therefore, its period and average z-direction drift velocity by simulation is 236 ns and 0.0066 mm/ns, respectively. The analytic results of the period and z-direction drift velocity from Eqs. (20) and (21) are 228 ns and 0.0063 mm/ns, respectively. They agree with the simulation results well. Note that a lower field gradient 0.5 T/m is used for this example. Figure 9 shows the distribution of the analytic τ and $\bar{\bar{v}}_{gz}$ of the trapped electron cloud as shown in Fig. 6(a). The peak of the τ distribution is close to two times that of the bunch spacing, which indicates the frequency of the interaction between the electron cloud and the positron bunch electric field. The peak of $\bar{\bar{v}}_{gz}$ distribution is around 3.5×10^{-3} mm/ns. The length of the quadrupole and sextupole magnets is 0.4 m for KEK LER. Therefore, the quadrupole and sextupole magnetic fields can strongly trap the photoelectrons for a very long time with level 10^5 ns until they drift out of the magnets in the beam direction. The revolution time of the positron beam around the accelerator is about 1×10^4 ns in KEKB LER. Therefore, most photoelectrons can be trapped for more than one revolution time. As a result, the trapped photoelectrons can cause multiturn effects to the positron beam.

The positron beam disturbs the photoelectron within a bunch train and hence changes the motion of the guiding center as shown in Fig. 8. First, F_v, and hence the reflective point, is modulated by the acceleration of the positron beam. Second, the positron beam also causes the photoelectron z-direction drift due to the $\mathbf{E}\times\mathbf{B}$ drift, where $\mathbf{E}$ is the beam electric field. Although the bunch length is short, the $\mathbf{E}\times\mathbf{B}$ drift is still important due to the strong electric field of the positron bunch (10^3–10^5 V/m). Therefore, comparing with the bunch train separation case, a photoelectron within bunch train has bigger drift velocity along the z direction.

V. CONCLUSION

A striking photoelectron-trapping phenomenon has been observed in our simulation study. The phenomenon is explained and agrees well with our analysis. Its mechanism is the mirror field trapping. The trapping is both magnetic field and beam dependent. There is no trapping for long bunch. The trapping time is long due to its very small z-direction velocity. Therefore, the trapped electron cloud can cause multitrain bunch interaction, even multiturn effects. Its effects on the beam should be studied in the future.

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